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Interpretation of cosmic ray muon data in the light of results from the intersecting storage ring experiment

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Abstract. The latest ISR data on p–p collisions at effective energies approaching 2×10^3 GeV have been used together with information on the primary spectrum to calculate the expected charge ratio of cosmic ray muons in the near vertical direction. It is found that the expected ratio is near to that observed for $E_\mu \simeq 30$ GeV without the necessity of assuming a breakdown in the validity of the limiting fragmentation hypothesis although the accelerator data are still not sufficiently precise to deny some small lack of validity. If the limiting fragmentation hypothesis continues to be valid at energies above 2×10^2 GeV and if the scaling hypothesis is applicable above 2×10^3 GeV, in particular if kaon production continues at the same rate, then it is necessary to assume that the relative flux of heavy nuclei in the primary cosmic ray beam increases somewhat with energy.

Previous analyses of the muon charge ratio are examined and the reasons for apparently discordant conclusions are indicated.

1. Introduction

Some of the present authors have recently examined the interpretation of the measured charge ratio of cosmic ray muons in the light of the data from the intersecting storage ring (ISR) experiment at CERN with regard to the information that can be gained about the difference between p–p and p–n collisions and about the primary mass composition. Thus, Hume *et al* (1973) concluded that straightforward application of the ISR data and the assumption of limiting fragmentation (to the extent that the energetic secondaries of p–p and p–n interactions are the same) resulted in a muon charge ratio somewhat higher than observed. They tentatively attributed this to a small breakdown of the fragmentation hypothesis, equivalent to strict applicability in 70% of interactions and non-applicability in 30%.

More recently Ng and Wolfendale (1974) have examined the variation of charge ratio with angle to the vertical and shown that the data suggest no dramatic change in the ratio of kaons to pions (K/π) and K^+/K^- over the energy range 5×10^2–10^4 GeV.

The analysis has also been extended to cover the examination of the mass composition of the primary cosmic rays at energies above those where direct measurements have been made. Daniel *et al* (1974) assumed the continuance of the scaling model for p–p interactions, which appears to have a measure of success at ISR energies, and showed that the measured muon charge ratios appear to indicate an increase in the fraction of heavy nuclei above about 5×10^2 GeV.

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In the present work a detailed examination is made of the problem using more accurate results from ISR and from primary composition measurements than were available hitherto. The ISR data refer to the energy range 3×10^2–2×10^3 GeV and strongly suggest that, for the majority of interaction products of p–p collisions of relevance here, the properties are not energy dependent from some tens of GeV to $1{\cdot}5\times10^3$ GeV. Essentially, the idea is to use the nuclear physical data and primary mass composition results, which are both precise below about 10^2 GeV, to examine the nuclear physics of nucleon–air nucleus interactions by comparing the predicted muon charge ratio with that observed. Then at higher energies, where neither nuclear physics nor mass composition is accurately known (ISR measurements extend to approximately 2×10^3 GeV but only give information about p–p interactions) we examine the consequences for each in turn if the other remains unchanged.

The actual method of calculation of the charge ratio is not described in great detail, instead attention is concentrated on the input data. In common with most other treatments, the results for primary nuclei are found by considering the superposition of collisions of the constituent nucleons. The results of other workers are also examined in detail in an effort to understand why the conclusions vary.

2. Experimental data from ISR

The ISR data of value to the present work relate principally to the inclusive cross sections for production of pions (π^+ and π^-) and kaons (K^+ and K^-) since it is from the decay of these particles that the muons are almost entirely derived. Ideally, measurements should be available for all transverse momenta (p_t) and fractional energies (x) but inevitably there are regions where interpolations and extrapolations must be made and this limits the accuracy with which predictions can be made.

Table 1 indicates the sources of the data used and figure 1 gives the results from the p_t distribution for particular values of x: 0·16 and 0·3. These values are chosen because

Table 1. Sources of data used in the present work.

Range of x	References for p–p and π–p interactions
$x \lesssim 0{\cdot}1$	Banner *et al* (1972, 1973), Alper *et al* (1973a, 1973b)
$0{\cdot}1 < x < 0{\cdot}3$	Bertin *et al* (1972), Albrow *et al* (1973a), Summary by Berger (1973)
$x > 0{\cdot}3$	Albrow *et al* (1973b, 1974): positive particles: Albrow *et al* (1973a): negative particles
all x	For $\pi^{\pm}$p interactions, accelerator data in the range 16–25 GeV/c were adopted: Law *et al* (1972)

the first is near the peak of the distribution of x values contributing to detected muons and the second a little above the average value. Before continuing, it should be pointed out that in the present work the symbol x is used for the ratio of the secondary particle energy to the incident proton energy in the laboratory system; conventionally it is the ratio in the centre-of-mass system but the two do not differ significantly for $x \gtrsim 0{\cdot}02$.

Distributions similar to those given in figure 1 have been derived from the experimental data for as wide a range of x values as possible and the differential cross section

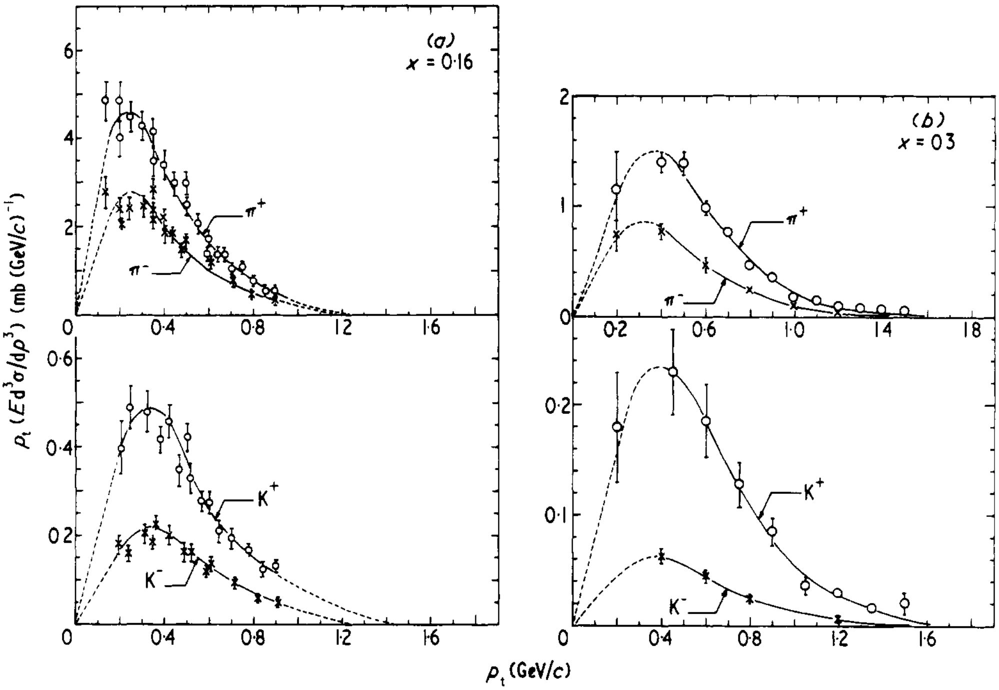

Figure 1. Dependence of invariant cross section for p–p interactions at ISR energies on transverse momentum for the most probable value of $x(x = 0{\cdot}16)$ contributing to the detected muons and for $x = 0{\cdot}3$. The sources of the data are referred to in table 1. The full lines are smooth curves drawn through the experimental points and the broken lines represent 'reasonable' extrapolations.

has been calculated:

$$\frac{d\sigma}{dx} = \int_0^\infty E\left(\frac{d^3\sigma}{dp^3}\right)\frac{\pi}{x}\, dp_t^2.$$

Values of $d\sigma/dx$ are given in table 2 for the various interactions contributing to the eventual production of muons and figure 2 shows $x\, d\sigma/dx$ plotted against x for π's and K's produced in p–p interactions.

Also shown in table 2 are moments needed later.

Figure 3 shows the dependence of $x\, d\sigma/dx$ on x for π^+ and π^- derived in the present work and given by previous workers in recent analyses (Adair *et al* 1973, Morrison and Elbert 1973). Of particular interest in the determination of the muon charge ratio is the ratio of cross sections for π^+ and π^-, $R_\pi(x)$, and this is shown in figure 4; again the results adopted by other workers are shown. The median value of x appropriate to the muon case is approximately 0·25 and, as remarked earlier, the corresponding most probable value is about 0·16; as will be seen later, the divergence between the various treatments for these values of x leads to an immediate explanation of at least some of the differences between the different calculations.

In the calculations to be described it is assumed, initially at least, that the effect of cascading in the nucleus is small. That is, the data for nucleon–nucleon 'collisions' are assumed to be applicable to the actual cosmic ray case of nucleon (and meson)–air nucleus collisions. This question has been examined recently by Wdowczyk and

Table 2. Adopted values of cross sections, $d\sigma/dx$ (mb), and other quantities, for secondary particles produced in p–p and π–p interactions.

process \ x	0·05	0·1	0·15	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	σ(mb)	$\langle nx^{1.6}\rangle$	$\langle nx^{1.7}\rangle$
$p\pi^+$	400	160	83·3	49	18·66	7·87	3·50	1·60	0·600	0·155	0·01	—	35	0·0411	0·0352
$p\pi^-$	370	130	62	31	10	4·0	1·70	0·75	0·314	—	—	—	35	0·0268	0·0226
pK^+	49·06	21·79	12·56	7·62	3·35	1·59	0·80	0·366	0·121	0·007	—	—	35	0·00730	0·00601
pK^-	31·54	12·16	4·82	2·50	0·80	0·319	0·13	0·034	—	—	—	—	35	0·00216	0·00181
$\pi^+\pi^+$	286	131·9	82	54·98	29·32	18·06	12·56	9·68	8·08	7·26	8·72	10·99	24·1	0·192	0·180
$\pi^+\pi^-$	163	78·5	47	29·84	13·61	6·67	3·77	2·20	1·39	0·98	0·66	—	20·0	0·0585	0·0520
$\pi^-\pi^+$	158	74·9	47	33·35	14·82	6·60	2·93	1·30	0·58	0·26	0·11	—	20·8	0·0472	0·0413
$\pi^-\pi^-$	270	128·8	80	53·4	27·23	16·49	11·31	8·38	7·18	7·46	10·12	10·99	25·3	0·178	0·168

Process: AB = A + p → B + anything

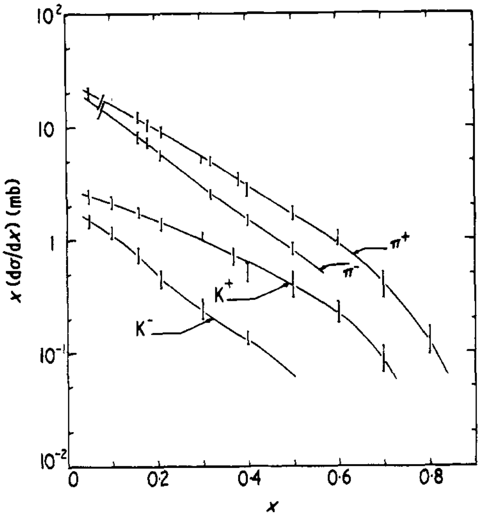

Figure 2. Differential inclusive spectra of secondary pions and kaons produced in p–p interactions at ISR energies. The error bars are derived from extreme fits to the contributing p_t dependences for fixed x (eg figure 1).

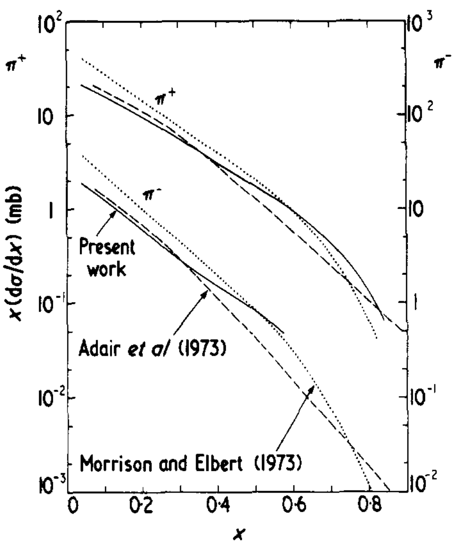

Figure 3. Differential inclusive spectra of secondary pions produced in p–p interactions at ISR energies: comparison of the spectra adopted by various authors. (The original data of Morrison and Elbert referred to p-air nucleus interactions, they have been scaled to correspond to p–p interactions by multiplying by the ratio of the cross sections, ie 35/275.)

Wolfendale (1973) and these authors concluded that the available information suggests that cascading is very small for the comparatively large x values ($x \gtrsim 0{\cdot}1$) relevant to the muon problem.

The ISR data refer to p–p collisions whereas information about p–p and p–n is required. The hypothesis of limiting fragmentation (Benecke *et al* 1969) indicates that

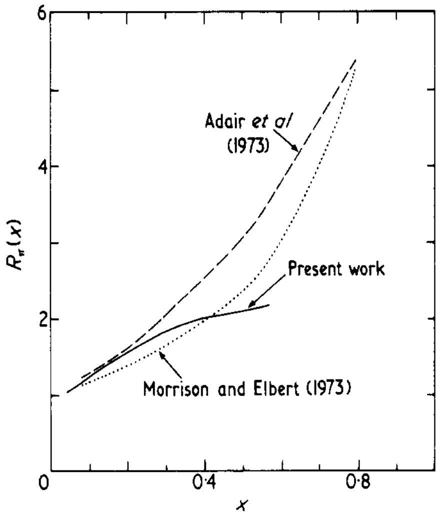

Figure 4. The ratio of cross sections for π^+ and π^- mesons, $R_\pi(x)$, plotted against x for ISR data on p–p collisions. Comparison of the derivations of various authors; that of Morrison and Elbert refers to p–nucleus rather than p–p interactions.

the resulting charge ratio for generated pions and kaons should be the same for $x \gtrsim 0{\cdot}1$ and, again, this will be assumed initially.

Finally, it will be assumed initially that the data for p–p (at ISR energies) and for π–p (in the tens of GeV region) are applicable at all energies. This assumption is in the spirit of the scaling hypothesis of Feynman (1969) and it appears to be borne out by accelerator experiments in the range 10–1000 GeV, at least for positive pion production. For negative pion production the approach to scaling is slower and full scaling appears to be reached only above about 70 GeV.

3. Primary composition measurements

A survey has been made of the fluxes of the main constituent nuclei in the primary beam with the result shown in table 3. The appropriate intensities of individual protons and neutrons are given, as is the aggregate ratio of neutrons to all nucleons of the same energy per nucleon, η. The survey includes the measurements of Ryan *et al* (1972), Smith *et al* (1973), Ormes *et al* (1973) and Grigorov *et al* (1970), the last mentioned referring only to energies above 100 GeV/nucleon.

At this stage it should be mentioned that there is evidence that the spectrum of iron is flatter than that of the other elements (Ormes *et al* 1973). It is with the possibility of this flattening continuing to higher energies than the present upper limit of measurement, about 100 GeV/nucleon, that we will be concerned later.

4. Method of calculation

The problem of calculating the expected energy spectrum of muons from a particular spectrum of primary nucleons has been considered by many authors. To sufficient

Table 3. Intensities of protons and neutrons in the primary cosmic radiation (values are given in units of particles/(m^2 s sr GeV/nucleon)).

Z, A \ E(GeV/nucleon)		10	20	30	50	100	200	300	500	1000
1	p	23·5	4	1·35	0·36	0·56	0·0085	0·0029	$7·1 \times 10^{-4}$	$1·15 \times 10^{-4}$
$A = 1$	n	—	—	—	—	—	—	—	—	—
2	p	3·0	0·505	0·166	0·043	0·006	$9·5 \times 10^{-4}$	$2·95 \times 10^{-4}$	$7·25 \times 10^{-5}$	9×10^{-6}
$A = 4$	n	3·0	0·505	0·166	0·043	0·006	$9·5 \times 10^{-4}$	$2·95 \times 10^{-4}$	$7·25 \times 10^{-5}$	9×10^{-6}
3–5	p	0·104	0·0155	0·0058	0·0015	2×10^{-4}				
$A = 6 \div 10$	n	0·104	0·0155	0·0058	0·0015	2×10^{-4}				
6	p	0·264	0·0432	0·0137	0·00318	$5·3 \times 10^{-4}$				
$A = 12$	n	0·264	0·0432	0·0137	0·00318	$5·3 \times 10^{-4}$				
7	p	0·077	0·0112	0·004	9×10^{-4}	$1·5 \times 10^{-4}$				
$A = 14$	n	0·077	0·0112	0·004	9×10^{-4}	$1·5 \times 10^{-4}$				
8	p	0·288	0·0576	0·0132	0·00304	$4·9 \times 10^{-4}$				
$A = 16$	n	0·288	0·0576	0·0132	0·00304	$4·9 \times 10^{-4}$				
9–14	p	0·402	0·0470	0·0151	0·00493	0·00128				
$\bar{Z} = 11, \bar{A} = 23$	n	0·438	0·0513	0·0165	0·00538	0·00140				
15–23	p	0·132	0·0320	0·0100	0·00328	0·0009				
$\bar{Z} = 18, \bar{A} = 40$	n	0·161	0·0391	0·0122	0·00401	0·0011				
$\geqslant 24$	p	0·116	0·0264	0·0111	0·0038	0·0008				
$\bar{Z} = 26, \bar{A} = 56$	n	0·134	0·0305	0·0129	0·0044	0·0009				
I(protons)		27·88	4·738	1·589	0·424	0·0664				
I(neutrons)		4·466	0·753	0·244	0·0654	0·0108				
$I^{\text{total}}_{\text{nucleons}}$		32·35	5·491	1·833	0·489	0·0771				
$\eta = \frac{I(\text{neutrons})}{I(\text{nucleons})}$		13·80%	13·72%	13·32%	13·37%	13·97%				

$\eta = (13·6 \pm 0·3)\%$; integral exponent, $\gamma = 1·62$.

accuracy the ratio of the vertical muon spectrum from pions generated in nucleon interactions, $M_\pi(E_\mu)$, to that of primary nucleons of the same energy, $N(E)$ is given by:

$$\frac{M_\pi(E_\pi)}{N(E)} = \frac{\lambda_\pi}{\lambda_p} A_\pi \langle nx^\gamma \rangle_{p\pi} \left(\sum_{m=1}^{10} \frac{(1-\lambda_\pi/L_p)^{m-1}}{1+(mr_\pi E_\mu/E_{0\pi})} \right) F(E_\mu)$$

(Bugaev *et al* 1970, Thompson 1973 and others).

The contribution from kaons, $M_K(E_\mu)$, is a similar expression with the subscript π replaced by K.

The terms involved in the expression and the adopted values are:

γ = slope of integral nucleon spectrum ($\gamma \simeq 1.62$ for $E_p < 10^3$ GeV and $\simeq 1.70$ above).
λ_p = interaction mean free path for nucleons
= 80 g cm^{-2} (the slow increase with energy found at ISR energies produces a negligible effect on the calculations).
L_p = attenuation length for nucleons = 110 g cm^{-2}.
λ_π = attenuation length for pions = 120 g cm^{-2}.
λ_K = attenuation length for kaons = 150 g cm^{-2}.
$\langle nx^\gamma \rangle_{p\pi(pK)}$ = fractional energy moments for pions (kaons) produced by nucleons—see table 2 (the value for kaons was multiplied by 0·62 to take into account only the muon decay mode).
$E_{0\pi}$ = critical energy for pion decay = 121 GeV; E_{0K} = 897 GeV.
A_π = kinetic constant = 0·691 for γ = 1·62
= 0·676 for γ = 1·70.
A_K = kinetic constant = 0·400 for γ = 1·62
= 0·390 for γ = 1·70.
$F(E_\mu)$ = factor to allow for muon decay and energy loss
$\simeq \left(1 - \frac{2.581}{E_\mu^{0.72}}\right)$ making an approximate fit to the numerical calculations of Osborne (1966).
r_π = 1·218 for γ = 1·62
= 1·216 for γ = 1·70
r_K = 1·386 for γ = 1·62
= 1·372 for γ = 1·70.

The numerical values have been taken from the work of Bugaev *et al* (1970).

For muons produced by pions arising from the interactions of secondary pions only first generation pions are considered. The solution of the kinetic equation gives:

$$\frac{M_{\pi\pi}(E_\mu)}{N(E)} = \frac{\lambda_\pi}{\lambda_p} A_\pi \langle nx^\gamma \rangle_{p\pi} \langle nx^\gamma \rangle_{\pi\pi} \frac{1}{1+(2r_\pi E_\mu/E_{0\pi})]} \left| 1 - b_{\pi\pi} \left(\frac{E_{0\pi}}{r_\pi E_\mu} \right)^{a_{\pi\pi}} \right| F(E_\mu)$$

(and the same for $\pi\bar{\pi}$ production) where

$a_{\pi\pi} = 0.421, \quad b_{\pi\pi} = 0.384$

$a_{\pi\bar{\pi}} = 0.579, \quad b_{\pi\bar{\pi}} = 0.271.$

The last term in brackets is the factor to allow for parent pion decay.

Muons are produced both in proton and neutron interactions but with different charge ratios and account must be taken of the difference in attenuation of protons and neutrons in the atmosphere. Expressing atmospheric depth as t (g cm^{-2}) the intensities of protons and neutrons are, respectively,

$$P(t) = \left(\frac{P_0+N_0}{2}\right)\exp\left(-\frac{t}{L_\mathrm{p}}\right)+\left(\frac{P_0-N_0}{2}\right)\exp\left(-\frac{t}{\Lambda}\right)$$

$$N(t) = \left(\frac{P_0+N_0}{2}\right)\exp\left(-\frac{t}{L_\mathrm{p}}\right)-\left(\frac{P_0-N_0}{2}\right)\exp\left(-\frac{t}{\Lambda}\right)$$

where P_0 and N_0 are the proton and neutron intensities at the top of the atmosphere ($N_0 = \eta(N_0+P_0)$).

$$L_\mathrm{p} = \frac{\lambda_\mathrm{p}}{(1-\langle\alpha^\gamma\rangle)}, \qquad \frac{1}{\Lambda} = \frac{1}{\lambda}(1+\langle\alpha^\gamma\rangle-2\beta\langle\alpha^\gamma\rangle)$$

where $\langle\alpha^\gamma\rangle$ is the elasticity moment and equals $(1-K)^\gamma$ where K is the effective inelasticity. β is the probability of the nucleon remaining in the same isospin state after collision.

The expressions given above have been used to calculate the expected intensities of positive and negative muons with the following further assumptions:

$$\langle nx^\gamma\rangle_{\mathrm{p}\pi^+} = \langle nx^\gamma\rangle_{\mathrm{n}\pi^-}, \qquad \langle nx^\gamma\rangle_{\mathrm{n}\pi^+} = \langle nx^\gamma\rangle_{\mathrm{p}\pi^-}$$

and

$$\langle nx^\gamma\rangle_{\mathrm{nK}^+} = \langle nx^\gamma\rangle_{\mathrm{nK}^-} = \tfrac{1}{2}(\langle nx^\gamma\rangle_{\mathrm{pK}^+}+\langle nx^\gamma\rangle_{\mathrm{pK}^-}).$$

The total spectrum of muons is given in the following paper and the ratio of number of positive to negative muons is considered in the next section.

5. Calculated muon charge ratio

5.1. Form of the expected variation

Figure 5 shows the calculated ratio for several variants of β and γ. The contribution from pions alone is also shown.

Although there is some increase in ratio expected for muons from pions alone the bulk of the increase arises from the effect of kaons. At this point it is relevant to note again that ISR data give information corresponding to $30 \lesssim E_\mu \lesssim 200$ GeV. At the bottom end of this range the primary mass composition is only just starting to become uncertain (the measurements on the important nuclei with $Z \gtrsim 10$ cease a little below 100 GeV/nucleon) and so the predictions for $E_\mu \simeq 30$ GeV should be rather firm.

Perhaps fortuitously the best estimate line passes through the experimental results in the region of 30 GeV and this would appear to suggest that the values of $\mathrm{d}\sigma/\mathrm{d}x$ at 'large' x for p–p and p–n interactions are the same, ie that the concept of limiting fragmentation is valid. However, it will be noticed that there is rather a large uncertainty in the predicted ratio, arising from the lack of precision, so far, of the ISR results. Indeed, as was mentioned in § 1, the earlier analysis of Hume *et al* (1973) using the ISR data then available gave rise to predicted ratios higher than observed and led to doubts on the strict applicability of the fragmentation idea. The most important change that has occurred since that work has been the extension of measurements of $\mathrm{d}\sigma/\mathrm{d}x$ beyond $x = 0{\cdot}3$, particularly for π^- production. As can be seen from figures 2 and 4, if the measurements are to be believed, there is evidence of a flattening off in the π^+/π^- ratio

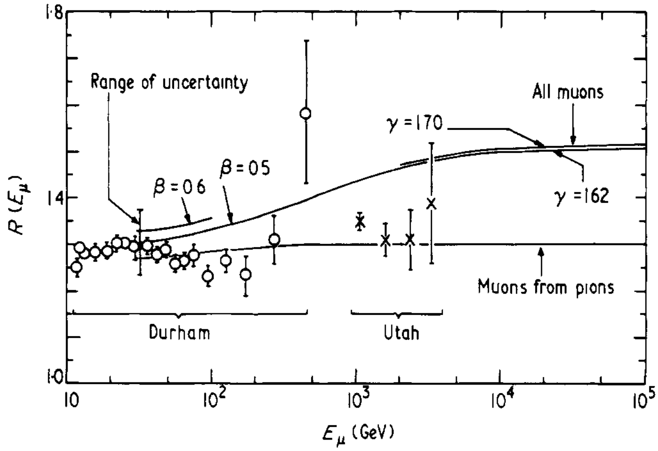

Figure 5. Comparison of observed and calculated charge ratio of near vertical muons $R(E_\mu) = |N(\mu^+)/N(\mu^-)|$. The sources of experimental data are: 'Durham', Ayre *et al* (1973); 'Utah', Morrison and Elbert (1973). The Utah data refer in fact to inclined directions ($51° < \theta < 74°$) and an upward correction of 0·03 has been applied to convert to $\theta = 0°$. The predictions distinguish between the ratio from pions alone with the overall ratio. Sensitivity to the integral spectral exponent γ and the probability of charge retention β is indicated. It is assumed that the primary mass composition is constant throughout.

as x increases beyond this value and this brings down the effective ratio from the 1·72 given by Hume *et al* to the value 1·54 adopted here.

It must be concluded then, that there is now no evidence for limiting fragmentation being invalid in the energy region of several hundred GeV, although more precise accelerator data are needed. Such data should be forthcoming shortly from the Batavia machine.

A further consequence of the agreement between experiment and theory based on p–p collisions is that there is no significant intranuclear cascading for the x values in question ($\simeq 0{\cdot}25$) (in fact some work has been reported by Baker *et al* (1974) for proton–beryllium interactions but this refers to a fixed angle of emission of the pions and kaons and does not cover the required range of x and p_+).

5.2. Possible consequences for nuclear physics

At muon energies above 30 GeV it is seen that there is increasing disagreement between prediction and observation. If the mass composition remains unaltered ($\eta = 0{\cdot}136$), as is assumed in the predictions of figure 5, then the nuclear physics of the interactions must change dramatically. Various possibilities arise:

(i) K^+/K^- falls with increasing energy. There is some evidence that the effective value of K^+/K^- falls between about 20 GeV and ISR energies (see Ng and Wolfendale 1974) but two facts indicate that this trend does not accelerate: over the ISR range of energy $250 \lesssim E_p \lesssim 1500$ GeV, K^+/K^- does not appear to change significantly and the analysis by Ng and Wolfendale of the variation of charge ratio with zenith angle suggests not much change for a further factor of about about 5 in E_p. However, it must be said that these arguments cannot be regarded as completely firm.

(ii) The probability of charge exchange rises with increasing energy, ie β falls. This is possible but appears most unlikely in view of the large change in β that is necessary.

(iii) The limiting fragmentation hypothesis becomes invalid above $E_p \sim 2 \times 10^2$ GeV. This could be connected with the suggestion that the scaling hypothesis may break down above 2×10^3 GeV with a faster than logarithmic increase in multiplicity of secondary particles (Wdowczyk and Wolfendale 1973). Such an increase might be expected to be accompanied by a fall in the π^+/π^- and K^+/K^- ratios, and a reduction in $R(E_\mu)$ for $E_\mu \gtrsim 200$ GeV, as required. Of the possibilities mentioned this is the most likely. The authors mentioned argue that at much higher energies than those relevant here (EAS energies) the effective multiplicity of secondaries may be much higher than expected on the scaling hypothesis although, as they point out, confusion is caused by uncertainty in the mass composition of the primaries. The question is, therefore, still an open one.

(iv) Onset of intranuclear cascading. Although there is no indication of cascading at $E_p \simeq 200$ GeV at the values of x and p_t relevant here it could conceivably become important at higher energies. However, such behaviour is at variance with common expectation, which is that cascading will be progressively less important as the interaction energy increases. It should be remarked that a theoretical analysis of cascading has been made recently by Lehman and Winbow (1974). These authors conclude that experimental data so far available are not sufficient for an adequate examination of the process.

An alternative explanation of the likely cause of the 'low' values of $R(E_\mu)$ is given in the next section.

5.3. *Possible consequences for the mass composition of primary cosmic rays*

From what has been said, and following the suggestion of Daniel *et al* (1974), constancy of the cross sections can be restored if the primary mass composition is allowed to vary appropriately. Figure 6 indicates the values of η which result from the measured muon charge ratios of figure 5 if the nuclear physics is assumed to be unchanged. In the figure the points are plotted at the median primary energies $\langle E_p \rangle_{med} \simeq 7{\cdot}1(E_\mu + 2)$ GeV (see the next paper).

The increase in η with E_p is rather marked and, if the analysis has any validity, there is the suggestion of a doubling of the fraction of neutrons in the primary beam by the time 10^4 GeV/nucleon is reached.

The possible increase is particularly interesting in view of the measurements of Webber *et al* (1965), Ormes *et al* (1973) (summarized by Ramaty *et al* 1973), and Juliusson *et al* (1972), referred to in § 3, that the iron spectrum has an exponent much smaller than that of protons. Specifically, the measurements indicate a value for the integral exponent γ_{VH} in the range 1·0 to 1·4 for energies in the range of measurement 4–40 GeV/nucleon. It may of course be that this exponent is subject to error or that there is a steepening in spectrum just above the energy range mentioned but it is instructive to extrapolate this flatter spectrum to much higher energies and to examine the consequences. Quite clearly the value of η will increase and will tend, eventually, to 30/56. Figure 6 shows the calculated variation of η with primary nucleon energy for the 'limiting' values of γ_{VH}: 1·0 and 1·4. In the calculation it has been assumed that only iron has the flatter spectrum; in fact, it is likely that other nuclei too show this feature, and if this is true η will rise more rapidly.

It is very interesting to note that the muon data are not inconsistent with $\gamma_{VH} = 1{\cdot}0$.

Daniel *et al* concluded that there was support for the low value of γ_{VH} up to about 200 GeV/nucleon, above which the normal composition was more appropriate. The

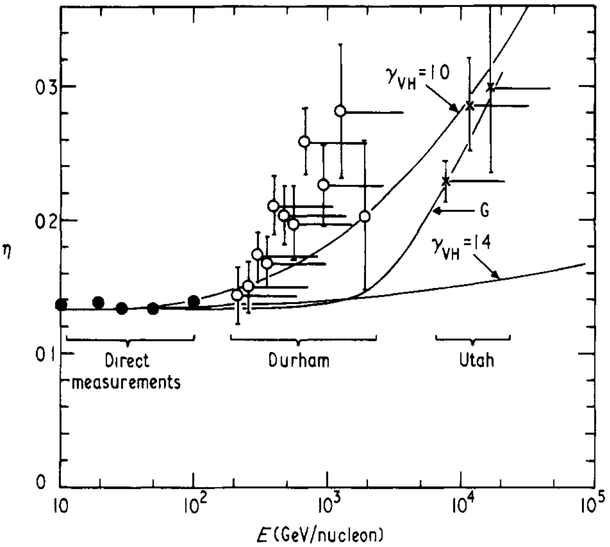

Figure 6. Ratio of neutrons to all nucleons, η, in the primary cosmic ray beam as a function of incident nucleon energy. The curves represent the situation if the compositions are as indicated. $\gamma_{VH} = 1{\cdot}0$, $1{\cdot}4$: very heavy nuclei have these integral exponents; intensities normalized to observation at 30 GeV/nucleon. G: constant composition for $Z > 1$ as at 30 GeV/nucleon with proton intensity falling rapidly above 10^3 GeV/nucleon, as indicated by the measurements of Grigorov *et al* (1970). The errors on the G curve are not shown; they are rather small compared with the difference between G and $\gamma_{VH} = 1{\cdot}0$ for $5 \times 10^2 < E < 5 \times 10^3$ GeV/nucleon. The experimental points are: 'direct measurements', the data of table 3; 'Durham' and 'Utah', derivations from the muon charge ratios of figure 5 assuming that scaling and limiting fragmentation persist to the highest energies. The horizontal lines join the open circles, plotted at the median primary energies, to the arithmetic mean primary energies ($\langle E_p \rangle \simeq 21(E_\mu + 2{\cdot}5)$ GeV).

present treatment suggests that perhaps above 10^3 GeV/nucleon the value of η is increasing less rapidly with energy than indicated by $\gamma_{VH} = 1{\cdot}0$, but a 'normal' composition is not allowed (figure 6). (By 'normal' is meant the composition at about 10 GeV/nucleon.) The reason for the discrepancy is that in the earlier work the contribution to the muon charge ratio from kaons was smaller than adopted here, because, effectively, smaller K^+/K^- ratios were used. We regard the present work as being more accurate.

As is well known, the PROTON satellite measurements (Grigorov *et al* 1970) suggest a rapid fall off in the proton spectrum above 10^3 GeV without much change in the spectrum of all nuclei. If this is correct, rather than the flattening of the iron spectrum, or no change in composition, then η will rise dramatically above 10^3 GeV/nucleon. Calculations of the form of η under these conditions have been made with the result shown in figure 6. It is apparent that the Utah results are quite consistent with expectation but the Durham data are not.

5.4. Conclusions about the interpretation of the charge ratio

Briefly, if the experimental measurements are correct then there is evidence that either the limiting fragmentation hypothesis ceases to be valid above several hundred GeV/nucleon or there is an increase in the relative number of nuclei other than protons in the primary cosmic ray beam above the same energy.

Table 4. Survey of results of recent calculations of expected muon charge ratio.

Reference	γ	η	(π^+/π^-) for p–p	μ^+/μ^- (E_μ = 30 GeV)	μ^+/μ^- (E_μ = 200 GeV)	Notes
Frazer *et al* (1972)	1·70	0·13	2·02	1·56	1·56	Used p–p data for E_p = 19·2 GeV which gives higher value π^+/π^- than at ISR in low p_t region.
Yekutieli (1972)	1·70	0·145	2·06	—	1·55	As above.
Morrison and Elbert (1973)	1·70	0·095	1·46	1·22	1·26	Used low energy p–nucleus data, which were available only for $x > 0{\cdot}3$ and give lower value of π^+/π^-.
Adair *et al* (1973)	1·70	0·24	1·83	—	1·46	Used rough approximation of ISR data which leads to higher values of π^+/π^- in the whole range of x. Very many neutrons.
Yen (1973)	1·70	0·13	1·70	1·40	1·40	Noticed that approach to scaling for π^+ and π^- is different and reduced π^+/π^- from 2·02 (Frazer *et al*) to 1·70.
Hume *et al* (1973)	1·60	0·11	1·72	1·28	1·30	π^+/π^- reduced to 1·44 for p–air to get measured μ^+/μ^- ratio.
Present result	1·62±0·02	0·136±0·003	1·54±0·15	1·31±0·07	1·36±0·07	See text.

6. Comparison with the results of other workers

It is necessary to examine the causes of discordant predictions of the muon charge ratio from the various treatments that have been made. Deviations between the results of the calculations can result from two causes: (i) different input data, and (ii) different models for propagation in the atmosphere having unequal degrees of accuracy.

Table 4 summarizes the more recent treatments, with particular regard to the important quantities η and the effective mean (π^+/π^-) ratio (ie $\langle nx^\gamma\rangle_{p\pi^+}/\langle nx^\gamma\rangle_{p\pi^-}$) derived from the accelerator data. Figures 3 and 4 also illustrate some of the differences. The reason for most of the differences in the final result can be seen directly.

The effect of differences in input data concerning the elasticity moments and the effect of (ii) referred to above can be observed by reference to the 'dilution factor', $D(E_\mu)$. This quantity is defined by the relation $\delta(E_\mu) = D(E_\mu)\,\delta_1(E_\mu)\,\delta p$ where $\delta(E_\mu)$ is the muon charge excess $\delta_1(E_\mu)$ is the excess expected for an incident beam of pure protons and δp is the proton excess $(= 1-2\eta)$. Figure 7 shows $D(E_\mu)$ from various authors, restricting attention to muons from pions. The most important parameters are the values of the elasticity moment, $\langle\alpha^\gamma\rangle$, and β, the probability of the nucleon remaining in the same isospin state after collision. It is more conventional to use the effective inelasticity K, $((1-K)^\gamma = \langle\alpha^\gamma\rangle)$ than the elasticity moment and values of this quantity are indicated in the figure. Inspection shows that there are differences in the values of K and β adopted and it is our view that they are not known to better than the range indicates. Furthermore, it will be apparent that even with very similar input parameters there are significant differences in derived dilution factors. It would appear that $D(E_\mu)$ is probably known to $\pm 5\%$ and, since $\mathrm{d}R(E_\mu)/R(E_\mu) \simeq 0{\cdot}35,\ \mathrm{d}\delta(E_\mu)/\delta(E_\mu)$ the fractional uncertainty in ratio from this cause is $\pm 1{\cdot}7\%$ ie about $\pm 0{\cdot}022$.

Clearly, the biggest change has been in the value of π^+/π^- ratio derived from accelerator experiments and although the data presently available are more precise than hitherto there is still room for improvement. Of particular interest are ISR measurements of π^+/π^- and K^+/K^- over a wide range of x values, but, most important, between

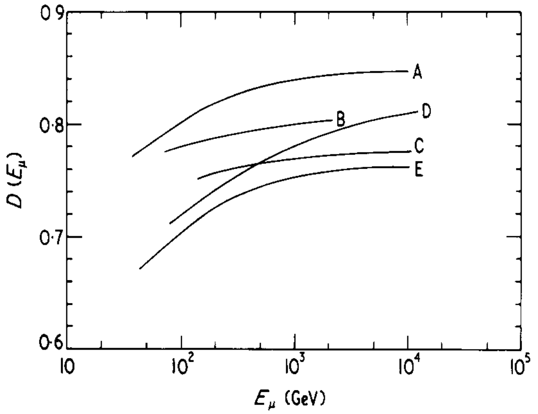

Figure 7. Comparison of dilution factors $D(E_\mu)$ from various calculations. β is the probability of charge retention. K is the effective mean inelasticity in p–p collisions. Curve A, present work ($K = 0{\cdot}55$, $\beta = 0{\cdot}5$); curve B, Hume (1974, private communication) ($K = 0{\cdot}45$, $\beta = 0{\cdot}565$); curve C, Hume (1974, private communication) ($K = 0{\cdot}45$, $\beta = 0{\cdot}5$); curve D, MacKeown and Wolfendale (1966) ($K = 0{\cdot}5$, $\beta = 0{\cdot}5$); and curve E, Morrison and Elbert (1973) ($K = 0{\cdot}46$, $\beta = 0{\cdot}57$).

0·2 and 0·6, and the probability distribution for the energy of the energetic proton or neutron. Concerning the problem of intranuclear cascading, measurements at NAL (E_p up to 400 GeV) on proton–nucleus interactions for various atomic masses will give useful information about the magnitude of cascading at these energies and will substantiate, or otherwise, the claim that such cascading (for 'large' x values: $\langle x\rangle \simeq 0{\cdot}25$) becomes less important at the higher energies.

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